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Bianchi Type-III String Cosmological Models

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Abstract Bianchi type III space time is considered in the presence of cosmic strings in Einstein's general theory of relativity. Exact cosmological models are presented with the help of relation $C = B^n$ between metric coefficients C and B. Some physical properties of the model in each cases are discussed.

Keywords Bianchi type-III · Cosmic strings · Relation between ρ and λ

1 Introduction

At the very early stage of evolution of the universe, it is generally assumed that during the phase transition the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain walls and monopoles [5]. Of all these cosmological structures, cosmic strings and domain walls have excited the most interest. In particular cosmic strings have becomes important in recent years from cosmological stand point. Bianchi type-III cosmological models are the simplest anisotropic universe models playing an important role in understanding essential features of the universe. In this class of models it is possible to accommodate the presence of topological defects (cosmic strings, domain walls, monopoles).

The study of cosmic strings has received considerable attention in cosmology since they play an important role in structure formation and evolution of the universe [4, 11]. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [11], Goetz [3], Letelier [7], Satchel [9] in general relativity. Relativistic string models in the context of Bianchi space times have been obtained by Krori et al. [6], Banerjee et al. [2], and Tikekar and Patel [10]. While Reddy and Subba Rao [8] have presented axially symmetric cosmic strings in Lyra geometry. Also, very recently Adhav et al. [1] have presented N-dimensional string cosmological model in Brans-Dicke theory of gravitation.

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The purpose of the present work is to obtain Bianchi type-III string cosmological models with the help of relation $C = B^n$ between metric coefficients. Our paper is organized as follows. In Sect. 2, we derive the field equations with cosmic string as a source with the aid of Bianchi type-III space time. Section 3 deals with some cases and it's solutions. The last section contains some conclusion.

2 Metric and Field Equatios

We consider the Bianchi type-III metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2, \tag{2.1}$$

where a is non zero constant. A, B, C are functions of time 't'.

The energy momentum tensor of cosmic strings in commoving coordinate system is given by

$$T^i_j = \rho u_j u^i - \lambda x_j x^i, \tag{2.2}$$

where ρ is the rest energy density of strings with massive particles attached to them and can be expressed as $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of the particles attached to the strings and λ is the tension density of the strings. Here u_i is the four velocity and x_i is the direction of string, obeying the relation.

$$u_iu^i = -x_ix^i = 1, \qquad u_ix^i = 0. \tag{2.3}$$

The cosmic string source is along z-axis which is axis of symmetry.

In commoving coordinate system, from (2.2), we have

$$T^1_1 = T^2_2 = 0, \qquad T^3_3 = \lambda, \qquad T^4_4 = \rho, \qquad T^i_j = 0 \quad \text{for } i \neq j. \tag{2.4}$$

The quantities ρ and λ depend on t only.

The Einstein's field equations are

$$R^j_i - \frac{1}{2}Rg^j_i = -8\pi T^j_i. \tag{2.5}$$

Now, with the help of (2.2), (2.3) and (2.4), the field (2.5) for the metric (2.1) reduces to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = 0, \tag{2.6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} = 0, \tag{2.7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{a^2}{A^2} = 8\pi\lambda, \tag{2.8}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4}{B}\frac{C_4}{C} - \frac{a^2}{A^2} = 8\pi\rho, \tag{2.9}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0. \tag{2.10}$$

From (2.10), we have

$$A = \mu B. \tag{2.11}$$

Using (2.11), (2.6) to (2.9) reduce to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = 0, \tag{2.12}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{\mu^2 B^2} = 8\pi\lambda, \tag{2.13}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4 C_4}{BC} - \frac{a^2}{\mu^2 B^2} = 8\pi\rho. \tag{2.14}$$

3 Some Cases with Solutions

The field (2.12) to (2.14) are three equations in four unknowns B, C, ρ and λ. Hence to get a determinate solution one has to assume a physical or mathematical condition.

In the literature there exists number of relations between ρ and λ, the simplest one being a proportionality relation given by

$$\rho = \alpha\lambda. \tag{3.1}$$

With the most usual choice of the constant α:
For

$\alpha = 1$ we get (geometric strings or Nambu strings)

$= -1$ we get (Reddy strings)

$= 1 + \omega$, $\omega \geq 0$ we get (p-strings or Takabayaski strings).

Case I: Geometric String or Nambu String ($\rho = \lambda$)

From (2.13) and (2.14), we get

$$\frac{B_{44}}{B} - \frac{B_4 C_4}{BC} = 0. \tag{3.2}$$

Here, assuming the relation between the metric coefficients as $C = B^n$, the field (2.12), (2.13), and (2.14) admit the exact solution given by

$$\begin{aligned} A &= L(k_1 t + k_2)^{\frac{1}{1-n}}, \quad \text{where } L = \mu M, \\ B &= M(k_1 t + k_2)^{\frac{1}{1-n}}, \quad \text{where } M = (1-n)^{\frac{1}{1-n}}, \\ C &= N(k_1 t + k_2)^{\frac{n}{1-n}}, \quad \text{where } N = M^n, \end{aligned} \tag{3.3}$$

$$8\pi\lambda = \frac{(2n+1)\,k_1^2}{(1-n)^2\,(k_1 t + k_2)^2} - \frac{a^2}{L^2\,(k_1 t + k_2)^{\frac{1}{1-n}}}. \tag{3.4}$$

The geometric or Nambu string cosmological model in Einstein's general theory of relativity corresponding to above solution can be written as

$$ds^2 = dt^2 - L^2 (k_1 t + k_2)^{\frac{2}{1-n}} dx^2 - M^2 (k_1 t + k_2)^{\frac{2}{1-n}} e^{-2ax} dy^2 - N^2 (k_1 t + k_2)^{\frac{2n}{1-n}} dz^2. \tag{3.5}$$

After a proper choice of coordinates and constants, (3.5) can be written as

$$ds^2 = \frac{dT^2}{k_1^2} - L^2(T)^{\frac{2}{1-n}} dX^2 - M^2(T)^{\frac{2}{1-n}} e^{-2ax} dY^2 - N^2(T)^{\frac{2n}{1-n}} dZ^2. \tag{3.6}$$

The physical parameters for the model (3.6) are

$$\text{Spatial volume} \quad v = \sqrt{-g} = \frac{LMN}{k_1}(T)^{\frac{1+n}{1-n}} e^{-ax} \tag{3.7}$$

$$\text{Scalar expansion} \quad \theta = \frac{(2+n)k_1}{(1-n)T} \tag{3.8}$$

$$\text{Shear scalar} \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\frac{(2+n)^2 k_1^2}{(1-n)^2 T^2}. \tag{3.9}$$

Case II: Reddy String ($\rho + \lambda = 0$)

From (2.13) and (2.14), we have

$$\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{B_4 C_4}{BC} = \frac{a^2}{\mu^2 B^2}. \tag{3.10}$$

Here again assuming the same relation, as in Case I, between the metric coefficients, the field equations (2.12), (2.13) and (2.14) admit the exact solution given by

$$\begin{aligned} A &= L_1(t+k_2), \quad \text{where } L_1 = \mu M_1, \\ B &= M_1(t+k_2), \quad \text{where } M_1 = \sqrt{\frac{a^2}{\mu(n+1)}}, \\ C &= N_1(t+k_2)^n, \quad \text{where } N_1 = (M_1)^n, \end{aligned} \tag{3.11}$$

$$8\pi\lambda = \left(1 - \frac{a^2}{\mu^2 M_1^2}\right)\frac{1}{(t+k_2)^2} = -8\pi\rho. \tag{3.12}$$

Reddy string cosmological model corresponding to above solution (3.11) will be given by

$$ds^2 = dt^2 - L_1^2 (t+k_2)^2 dx^2 - M_1^2 (t+k_2)^2 e^{-2ax} dy^2 - N_1^2 (t+k_2)^{2n} dz^2. \tag{3.13}$$

After the proper choice of coordinates and constants, (3.13) can be written as

$$ds^2 = dT^2 - L_1^2 (T)^2 dX^2 - M_1^2 (T)^2 e^{-2ax} dY^2 - N_1^2 (T)^{2n} dZ^2. \tag{3.14}$$

The physical properties for the model (3.14) are

$$\text{Spatial volume} \quad \upsilon = \sqrt{-g} = (L_1 M_1 N_1) T^{n+2} e^{-ax} \tag{3.15}$$

$$\text{Scalar expansion} \quad \theta = \frac{n+2}{T} \tag{3.16}$$

$$\text{Shear scalar} \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left(\frac{n+2}{T}\right)^2. \tag{3.17}$$

Case III: p-String or Takabayaski String: $\rho = (1+\omega)\lambda$

From (2.13) and (2.14), we have

$$2(1+\omega)\frac{B_{44}}{B} + \omega\left(\frac{B_4}{B}\right)^2 - \frac{2B_4C_4}{BC} = \frac{\omega a^2}{\mu^2 B^2}. \tag{3.18}$$

Using earliar relation, the field equations (2.12), (2.13) and (2.14) admit the exact solution given by

$$\begin{aligned} A &= L_2(t+k)^{\frac{1}{2}}, \quad \text{where } L_2 = \mu M_2, \\ B &= M_2(t+k)^{\frac{1}{2}}, \quad \text{where } M_2 = \frac{\sqrt{2}wa^2}{-\mu^2(2n+2+\omega)}, \\ C &= N_2(t+k)^{\frac{2}{n}}, \quad \text{where } N_2 = (M_2)^n, \end{aligned} \tag{3.19}$$

$$8\pi\rho = \left(\frac{n+8}{4n}\right)\frac{1}{(t+k)^2} - \frac{a}{\mu^2 M_2^2(t+k)}, \tag{3.20}$$

$$8\pi\lambda = \frac{-a^2}{\mu^2 M_2^2(t+k)}. \tag{3.21}$$

The p-string model corresponding to above solution (3.19) can be written as

$$ds^2 = dt^2 - L_2^2(t+k)\,dx^2 - M_2^2(t+k)\,e^{-2ax}dy^2 - N_2^2(t+k)^{\frac{4}{n}}\,dz^2. \tag{3.22}$$

After the proper choice of coordinates and constants, the (3.22) can be expressed as

$$ds^2 = dT^2 - L_2^2(T)\,dX^2 - M_2^2(T)\,e^{-2ax}dY^2 - N_2^2(T)^{\frac{4}{n}}\,dZ^2. \tag{3.23}$$

The physical parameters of the model (3.23) are

$$\text{Spatial volume} \quad \upsilon = \sqrt{-g} = (L_2 M_2 N_2) T^{\frac{n+2}{n}} e^{-ax} \tag{3.24}$$

$$\text{Scalar expansion} \quad \theta = \left(\frac{n+2}{n}\right)\frac{1}{T} \tag{3.25}$$

$$\text{Shear scalar} \quad \sigma^2 = \frac{1}{6}\left(\frac{n+2}{n}\right)^2\frac{1}{T^2}. \tag{3.26}$$

Discussion

The energy density and tension density of the strings are given by (3.4), (3.12), (3.20), (3.21) respectively. It may be observed that at initial moment, when $T = 0$, the spatial volume will be zero while the rest energy density ρ and tension density λ diverges. When $T \to 0$, then expansion scalar θ and shear scalar σ^2 tends to infinity. For large values of T $(T \to \infty)$ we observe that spatial volume tends to infinity, while expansion scalar θ, shear scalar σ^2, rest energy density and tension density becomes zero.

Also $\lim_{T\to\infty}(\frac{\sigma}{\theta})^2 \neq 0$ and hence the model does not approach isotropy.

4 Conclusion

In this paper, we have obtained Bianchi type-III cosmological models in the presence of cosmic string source which corresponds to geometric string, Reddy string and p-string. It is observed that the models in all cases are similar and behave alike. The cosmic string models studied here will be useful for better understanding of cosmology and structure formation of the universe.

References

1. Adhav, K.S., Nimkar, A.S., Dawande, M.V.: Astrophys. Space Sci. **310**, 231–235 (2007)
2. Banerjee, A., Sanyal, A., Chakraborty, J.: Pramana J. Phys. **34**, 1 (1990)
3. Goetz, G.: J. Math. Phys. **31**, 2683 (1990)
4. Kibble, T.W.B., Turok, N.: Phys. Lett. B **116**, 141 (1982)
5. Kibble, T.W.B.: J. Phys. A **9**, 1387 (1976)
6. Krori, K.D., Choudhari, T., Mahanta, C.R., Mujumdar, A.: Gen. Relativ. Gravit. **22**, 123 (1990)
7. Letelier, P.S.: Phys. Rev. D **28**, 2414 (1983)
8. Reddy, D.R.K., Subba Rao, M.V.: Astrophys. Space Sci. **302**, 157–160 (2006)
9. Satchel, J.: Phys. Rev. D **21**, 2171 (1980)
10. Tikekar, R., Patel, L.K.: Gen. Relativ. Gravit. **24**, 397 (1990)
11. Vilenkin, A.: Phys. Rev. D **23**, 852 (1981)